LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2023**

PMT 4505 – CLASSICAL MECHANICS

Date: 08-05-2023 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

Answer ALL the questions

1. (a) Define (i) angular momentum (ii) torque (iii) virtual displacement. (5)

(OR)

(b) Write down the Lagrangian equation, if the Lagrangian has the form

$L = -(1 - \dot{q}^2)^{1/2}$. (5)

(c) (i) State and prove D'Alembert's principle.

(ii) If the torque for the moment of external force of the system about a point O is zero then prove that the angular momentum about that point is conserved. (5+10)

(OR)

(d) Derive the Lagrange's equation of motion for a holonomic system and hence deduce it for the conservative system. (15)

2. (a) Obtain Hamilton's canonical equations from Hamilton's principle. (5)

(OR)

(b) Given the following Lagrangian for a harmonic oscillator, find its corresponding Hamiltonian: $L(x, \dot{x}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}w^2x^2 - \alpha x^3 + \beta x\dot{x}^2$ where α, β and w are constants. (5)

(c) If the component of the applied torque along the axis of rotation vanishes then prove that the component of total angular momentum along the axis of rotation is conserved. (15)

(OR)

(d) Write a brief note on Routh's procedure and Legendre transformation. (15)

3. (a) Prove that Poisson bracket is invariant under canonical transformation. (5)

(OR)

(b) Derive Jacobi's form of the principle of least action. (5)

(c) State and prove Poincare theorem. (15)

(OR)

(d) State and prove the different possibilities for the generating function in canonical transformation. (15)

4. (a) Derive the transformation equation for infinitesimal contact transformation in terms of Poisson bracket. (5)

(OR)

(b) Derive equations of motion in Poisson bracket. (5)

(c) State and prove Liouville's theorem. (15)

(OR)

(d) Derive the Angular momentum - Poisson bracket relation. (15)

5. (a) Write a brief note on the types of periodic function. (5)

(OR)

(b) Find the action-angle variable for Simple Harmonic Oscillator. (5)

(c) Discuss the Kepler's problem using Action Angle Variable. (15)

(OR)

(d) Derive the solution for Simple Harmonic Oscillator problem by Hamilton Jacobi method. (15)

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